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BOOK OF ABSTRACTS

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Non-Local Theory of Plasmoids. Problem of the Tunguska Explosion. Gagarin Catastrophe Version

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Non-local statistic theory of dissipative processes is created by methods of non-local physics. Theory is applicable for the physical system description in giant diapason of scales – from the structure of so called "elementary particles" and nuclei to the Universe scale including the inflation period. All local statistical theories of non-equilibrium processes are wrong in principle (Bell's theorems).

The theory (in the frame of the local asymptotic approximation of non-local equations) is applied for the investigation of internal structures of single physical systems with the separated charges. The theory leads to solitons as typical formations in the generalized quantum hydrodynamics. The domain of the soliton existence is working out as result of the self-consistent solution of equations. The existence in a bounded region of the self-consistent plasma objects is established by the methods of non-local physics. The unified generalized non-local theory is applied for mathematical modeling of quantum solitons.

Lightning balls are quantum solitons – plasmoids which reach stability as result of equalizing of corresponding quantum pressures of the non-local origin and the self-consistent electric forces. The delivered theory demonstrates the great possibilities of the generalized quantum hydrodynamics in investigation of the quantum solitons.

Two catastrophes of 20th century – Tunguska explosion and Gagarin catastrophe have the same physical origin – plasmoid appearance in the Earth atmosphere. The explanation of Gagarin catastrophe is delivered as the result of the interaction of MIG 15 UTI with plasmoid.

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Solution of Falkner-Skan Equation with Heat Transfer Using a Legendre Collocation Method

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In this paper, we introduce a new spectral collocation method for solving the Falkner-Skan equation with heat transfer. The Falkner-Skan equation is a nonlinear third-order boundary value problem on semi-infinite domain. The properties of the Legendre polynomials are used to approximate an unknown function and its integrals. Then the shifted Legendre-Gauss points are utilized as the collocation points to reduce the problem to the solution of an algebraic system. The results show that the proposed method delivers a solution with high accuracy and very rapid convergence. Comparisons with other methods in the literature demonstrate the applicability and accuracy of the proposed method.

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On the Excitation of Goodwin's Oscillations

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We have considered the condition of excitation of long-period Goodwin's oscillations for the following model of the economic cycle [1]

$$\varepsilon\dot{y}(t) + (1-\alpha)y(t) = \varphi(\dot{y}(t-\theta)).$$

Here $y(t)$ is income, $\varepsilon > 0$ and $\theta > 0$ are the time-lag of the dynamic multiplier and the time-lag between investment decisions and the resulting outlays, respectively,

α is the marginal propensity to consume, $0 \le \alpha \le 1$, φ is the investment induced by the change in income. The numerical investigation of this problem was carried out partially in [2].

To obtain this condition we have used the method of equivalent linearization [3]. The condition of Goodwin's oscillations excitation is written as

$$r > \frac{\varepsilon}{\cos u}$$

where $r = \frac{d\varphi(0)}{d\dot{y}}$, u is a root of the equation

$$u \tan u = \frac{(1-\alpha)\theta}{\varepsilon}, \quad 0 < u < \frac{\pi}{2}.$$

If $\varepsilon < r < \frac{\varepsilon}{\cos u}$ only the sawthooth oscillations with periods 1, 1/2, 1/3 ... are excited. For $\varepsilon > r$ the solution $y(t)$ approaches the stationary point $y_s = 0$ as shown in [4].

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Soliton Solutions for Tzitzeica II Equation

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The paper investigates the soliton solutions of Tzitzeika II equation, similar to Tzitzeica I equation proposed more than a century ago for the analiyes of special surfaces in differential geometry and important as an example of completely integrable 2-dimensional Toda field theory.

We use the dressing Zakharov-Shabat method for deriving the soliton solutions, both the two versions of Tzitzeika equations. The dressing factor is written as a rational function of the spectral parameter and we investigate two classes of soliton solutions: one corresponding to dressing factor with three poles and a second class, when the dressing factor has 6 poles. We establish that for both equations the poles of the dressing factors and their inverse are discrete eigenvalues of the Lax operator. Besides, while the soliton solutions of the first equation have singularities, the soliton solutions of Tzitzeika II are regular and localized.

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Numerical Investigation of the Impact of Seismic Waves on the Elastic-Plastic Structures Buried in the Ground

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We consider the problem of dynamic loading for various external impacts of elastic-plastic structures and using the computing resources of modern e-infrastructure to solution and analysis of such problems. These problems have real practical importance. The existing structures are often filled with flammable, explosive or aggressive substances. Often is necessary to ensure the functioning of these constructions at high loads (earthquakes, explosions, and other cases of force majeure). Numerical simulation of seismic or blast waves propagation in different grounds and analysis of their impact on the buried structures is particularly interesting for areas with high seismic activity (including Moldova). The mathematical model, the system of determining equations and the results of numerical studies of the buried elastic-plastic structure behavior under the seismic (or shock) waves loading are presented.

The proposed mathematical model of loading takes into account the propagation of waves in the elastic-plastic structure and in the filling medium.

The pressure $P = f(t)$, existing at some distance from the seismic source, is calculated for the case of a seismic load. A wave spectral model (the Mueller-Murphy model) $P(t) = (P_0 e^{-dt} + P_{oc})H(t)$ was chosen, where $P_0 + P_{oc} = P_{os}$ is the peak impact pressure, P_{oc} is the settled pressure, d – is the attenuation constant, $H(t)$ is the Heaviside step function [1]. The ground we consider as a ternary medium (solid-water-air). State equation for the ground under intense loading may be presented as [2].

Let us introduce respectively gaseous, liquid and solid components for the ternary soil. Moreover, the following relationship occurs; are isentrope indicators for the corresponding components; are the values of density; – sound velocity corresponding to the gaseous, liquid and solid components. The ground density at has the form . The software product for studying and prediction of operational status of elastic-plastic systems in a wide range of parameters of structural materials and external influences was developed that is based on the proposed principles of mathematical modeling and methods of numerical calculations.

We propose here an extension of Wilkins method for the study of elastic-plastic deformation of structural elements [3, 4]. Five equations of state are used for modeling of the behavior of elastic-plastic construction materials as well as water and ground. The numerical results for specific structures reflect a real picture of the interaction of the ground and buried construction object under impact of explosive or seismic wave load. For dynamic visualization of the obtained results has been developed the special software module written on OpenGL programming language. The rapid development of information technologies contributes to the creation of high-performance computing environments as a powerful tool for solving a wide range of practical tasks by using various methods of computer modeling. In the paper we describe the accumulated experience of development and use of computer infrastructures and services for mathematical modeling of the proposed problems that are requiring large amounts of computing resources [5].

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Euler-Lagrange Active Contour Evolving in a Poison Vector Field of an Image

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This paper presents a new active contour to segment color images. The model uses the solution of the Euler-Lagrange Differential Equation in order to minimize a "snake" functional [4]. This functional is constructed as an integral of the so called internal and external energies [3]. The internal energy is related to the moving contour. The external energy represents the image. The minimum of the functional falls on the boundaries of objects placed in the image [2,4]. A half step numerical scheme implements the concepts [4]. In order to make the contour move across homogeneous regions, we solve the Poisson equation with Dirichlet boundary conditions and generate a gradient vector field of the image [1]. This vector field moves the contour toward the objects boundaries and halts the contour on the boundaries with the help of penalty function which employs the image gradient [1]. A numerical method has been developed on the basis of the theoretical concepts and implemented with MatLab. The advantages of the model are that it has a large capture range, is accurate in detecting image objects boundaries, and capable of surpassing noise [1]. A disadvantage is that the user has to select the right values of three parameters. Several experiments with synthetic and medical images have been conducted to validate the model. Our work continues with the replacement of one of the parameters with the contour's curvature. The goal is to cut the curve in the event multiple objects have been enveloped.

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Two-Axes Decompositions of (Pseudo-)Rotations and Some of Their Applications

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We propose a convenient method for decomposing $\mathsf{SO}(3)$ transformations into pairs of successive rotations (about orthogonal axes) and consider some applications in rigid body kinematics and navigation. The construction is then spread to $\mathsf{SO}(2,1)$ by means of Wick rotation (duality of Lie algebras) and $\mathsf{SO}(3,1)$ via complexification of the vector parameter [1,2,3,4]. Possible applications of the Lorentz setting are discussed in the context of special relativity, electrodynamics and scattering theory.

Key words and phrases: Vector parameters, orthogonal decomposition, rigid body kinematics, special relativity, electrodynamics, scattering theory

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Frequency-Dependent Performance Analysis of a Parallel DSP-Based Computer System

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A shared-memory DSP-Based multiprocessor system was proposed by the author in a previous published work. The innovation of the proposed architecture was the introduction of two small programmable fast memories (Twins) between the processor and the shared bus interconnect. While one memory (Twin) transfers data from/to the shared memory, the other Twin supplies the core DSP-processor with data. In this paper, the performance of the simulated parallel architecture is examined, by varying the frequency of the core processor and keeping constant other parameters such as the number of DSPs, the Twin's cache-memory size, the shared bus width and the shared-memory-access-time. Results indicate greater shared-bus bottleneck as the core DSP processors' clock-rate increases. Workload of the Twins is processed faster thus greater the demand of the shared-bus.

Keywords: DSPs, prefetching, shared bus, shared memory, bottleneck

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Thermoacoustic Waves in a Cylindrical Couette Rarefied Gas Flow

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The thermoacoustic waves arising in cylindrical rarefied gas Couette flow between rotating cylinders is studied in the cases of a suddenly stop or suddenly rotation direction turn on the one or the two active cylinders. Based on the developed in previous publications Navier-Stokes-Fourier (NSF) model and Direct Simulation Monte Carlo (DSMC) method and their numerical solutions, are considered non-stationary transients process in the gas phase. Macroscopic flow characteristics (velocity, density, temperature) and the viscous and thermal interaction of the gas with the walls are received. The typical cases involving different combinations of the kinematic boundary conditions on the walls are considered. The wave?s distribution and attenuation are studied numerically.

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On Cyclic Asymptotic Regularity and Approximate Best Proximity Points

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The paper studies cyclic self-mappings from a union of subsets to itself in metric spaces to the light of asymptotic regularity properties. In that context, approximate best proximity points are investigated and characterized. In the case that the domain and image sets of the self- mappings intersect, the above study relies on approximate fixed points. Mathematical results are obtained related to this research.

Keywords: Approximate best proximity points, approximate best proximity property, approximate partial best proximity property, cyclic self-maps, cyclic asymptotic regularity

The paper studies cyclic self- mappings from a union of subsets to itself in metric spaces to the light of asymptotic regularity properties. In that context, approximate best proximity points are investigated and characterized. In the case that the domain and image sets of the self- mappings intersect, the above study relies on approximate fixed points. Mathematical results are obtained related to this research.

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Numerical Approach for an Optimal Harvesting Policy in a Diffusive Predator-Prey System

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An optimal control problem associated with a diffusive predator-prey system with Holling type II functional response and no-flux boundary condition is considered. The parameters representing the harvesting rates for predator and prey populations are treated as multiplicative control variables. The target is to maximize the revenue of the harvesting and to minimize the cost of the controls. Such models present a wide-range of interest in the use of bioeconomic modeling and species conservation to gain insight in the scientific management of renewable resources like fisheries and forestry. Numerical results of the optimal solution using different scenarios are presented.

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A New Walk on Equations Monte Carlo Method for Linear Algebraic Problems

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A new Walk on Equations (WE) Monte Carlo algorithm for Linear Algebra (LA) problems is proposed and studied. This algorithm relies on a non-discounted sum of an absorbed random walk. It can be applied for either real or complex matrices. Several techniques like simultaneous scoring or the sequential Monte Carlo method are applied to improve the basic algorithm. Numerical tests are performed on examples with matrices of different size and on systems coming from various applications. Comparisons with standard deterministic or Monte Carlo algorithms are also done.

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The In-phase States of Josephson Junctions Stacks as Attractors

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The main purpose of this investigation is to study the attractors of the states of the intrinsic Josephson junctions stacks depending on two parameters, that can be controlled in the real experiments – the external magnetic field h and the external current γ. Mathematically the problem is to find the solutions of the system of perturbed sine-Gordon equations for fixed other parameters and zero or random initial conditions. The aim is to determine the region in the plane (h, γ), where the in-phase states are attractors of the stack's states for arbitrary initial perturbations. This is important, because the in-phase states are required for achieving terahertz radiation from the Josephson stacks.

This time-consuming study with respect to the parameters can be separated into independent tasks in the Grid computing infrastructure. We use the computational cluster of IICT-BAS, which is part of the European Grid infrastructure (EGI). The process of "sending" the jobs to the Grid and "collecting the output" results is fully automated and implemented on the Grid user interface of the IICT-BAS.

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Evaluation of Collisional Ionization Rate Constants in Discharge Plasma by the Optogalvanic Effect

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Laser optogalvanic spectroscopy is a simple and sensitive detection technique based on the optogalvanic effect. This effect represents impedance change of gaseous discharge plasma due to the absorption of resonant laser light by the discharge species. Absorbed light changes the steady-state population of the energetic levels involved in the illuminated optical transition and thuse affects the effective ionization processes in the plasma. Laser optogalvanic spectroscopy has various applications for wavelength calibration, trace elelement detection, isotope analysis, laser frequency and power stabilisation, line width determination, Rydberg atom spectroscopy and plasma diagnostics. The relative magnitudes, signs, and time evolution of the optogalvanic signals contain information for generation of excited and ionized particles ant their quenching during the time. It can be explained qualitatively but detailed quantitative description of the optogalvanic effect is perhaps difficult mainly because it is impossible to give a complete set of rate equations for all the levels and processes involved in the discharge.

In the present work the dynamic optogalvanic effect from 1s4-2p10 (540.05nm) and 2p10-4d3 (533.08nm) neon atomic transitions and the associated time-resolved optogalvanic signals in a hollow cathode discharge has been investigated. System of rate equations including the processes associated with the generation of the optogalvanic signals is developed. The dominant plasma process responsible for optogalvanic signal generation is identified. The decay rate constants of the levels contributing to signal formation have been determined using non-linear fit of the theoretically obtained function with experimentally measured optogalvanic signals. The behavior of the time resolved optogalvanic signal waveform, together with the decay rate constants as a function of the discharge current have been analyzed. The procedure for deconvolution of registered optogalvanic signals with apparatus function is also presented. The approach described here is useful in evaluation of ionization rates of electronically excited states from time resolved optogalvanic signal measurements and determining of the processes responsible for optogalvanic signal generation.

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Controllability Problem for the Solution of the String Equation with an External Load with Fixed Ends on a Rectangular

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Controllability problems for partial differential equations are being investigated nowadays by a number of mathematicians (J.-L. Lions, V.A. IlЃfin, E. Zuazua, L.V. Fardigola, M. Gugat, Ye.R. Ashrafofa, etc.).

But in some cases the controls introduced in the boundary conditions can not be realized from a practical point of view. In the article we build the controls introduced in the function of an external load, which solve the problem of approximately null-controllability and nullcontrollability.

We complete the investigation of the controllability problem for the wave equation

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} + g(x,t), \quad x \in (0,\pi),\ t \in (0,T),\ T > \pi \tag{1}$$

$$g(x,t) = \sum_{j=1}^{n-1} \delta(x - x_j) \sum_{k \in Z} \mathfrak{T}_{2Tk} g_j^n(t) \tag{2}$$

where $\delta(x)$ is the Dirac function; $\mathfrak{T}_h$ is the translation operator: $(\mathfrak{T}_h \varphi)(x) = \varphi(x + h, t)$. Provided $x_j = \frac{\pi j}{n}$, $j = \overline{1,n}$, $n \in \mathbb{N}$, $t \in (0,T)$, $T > \pi$, and $g_j^n(t)$ is the control, $j = \overline{1,n}$, $n \in \mathbb{N}$.

The initial conditions

$$w(x,0) = \frac{\partial w(x,0)}{\partial t} = 0, \quad x \in (0,\pi),\ t \in (0,T). \tag{3}$$

We have completed the investigation of the controllability problem for the solution of the wave equation with an external load.

The criterion of approximately null-controllability for system (1)-(3) with the restriction of control is obtained. The controls solving these problems are found explicitly.

In this paper necessary and sufficient conditions for null-controllability and approximate null-controllability are obtained for the solution of the string equation with fixed ends controlled by the external load on the rectangular.

The criterion of null-controllability for the solution of the string equation was obtained.

Obtained results in the future may be serve as a basis for solving the controllability problems in the study of the wave equations of higher dimensions with external load and without it. That is, when considering solutions on the plane, in space.

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Improved Boundary Condition Approximation for Accurate Numerical Solution of PDEs

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Numerical methods like FEM, FDM, RBF and others are successfully used for simulations of a wide range of problems in science and technology. Numerical treatment of boundary conditions is very important in case of thin boundary layers for the solution at the boundary, and naive mesh refinement may not help, resulting in the unacceptable $O(1)$ approximation of the solution, as showed by Shishkin in [1]. Shishkin has shown that there is a sophisticated mesh improvement, but this may result only in $O(h^{1/m})$ $(m \geq 7)$ approximation of the solution.

In this work we propose a non-traditional approach, which may result in a dramatic solution accuracy improvements, sometime by three of for orders of magnitude. This approach first was applied to the solution of PDEs by Multiquadric (MQ) Radial Basis Function (RBF) method [2]. The multiquadric radial basis function (MQ) method is a recent meshless collocation method with global basis functions. It was introduced for discretizing partial differential equations (PDEs) by Kansa in the early 1990s. The MQ method was originally used for interpolation of scattered data, and it was shown to have exponential convergence for interpolation problems. In [3], we have extended the Kansa-MQ method to numerical solution and detection of bifurcations in 1D and 2D parameterized nonlinear elliptic PDEs. We have found there that the modest size nonlinear systems resulting from the MQ discretization can be efficiently continued by a standard continuation software, such as AUTO. We have observed high accuracy with a small number of unknowns, as compared with most known results from the literature. We formulated an improved Kansa-MQ method with PDE collocation on the boundary (MQ PDECB), by adding an additional set of nodes (which can lie inside or outside of the domain) adjacent to the boundary and, correspondingly, add an additional set of collocation equations obtained via collocation of the PDE on the boundary [4]. Numerical results are given that show a considerable improvement in accuracy of the MQ PDECB method over the Kansa-MQ method, with both methods having exponential convergence with essentially the same rates.

The details of the proposed approach will be presented for a number of methods, including the finite element method, and numerical solutions will be presented, demonstrating dramatically improving resolution of the boundary layer on a few nodes.

Keywords: Nonlinear elliptic PDEs, Numerical Solutions, Boundary Condi-

tion Approximation, Radial basis functions, Finite Element Method

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Ultra-Efficient Solar-Energy-Conversion Pathway with Dilute Nitride Quantum-Enhanced Photovoltaics

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Development of III-V multijunction devices incorporating $1 - 1.1$eV dilute nitride subcells has been to a large extent limited by poor minority carrier diffusion lengths encountered in bulk dilute nitrides which in turn have led to severe degradations of device open circuit voltages.

Previously we have shown that an alternate design, where dilute nitride quantum wells are inserted within the intrinsic region of a conventional GaAs p-i-n subcell could be used to overcome these limitations and significantly improve the open circuit voltages compared to bulk-counterparts [1,2]. Furthermore the negligible valence band offsets and the unusual increase of electron effective masses in dilute nitrides provide a somewhat unique opportunity to exploit aperiodic/asymmetric quantum well designs that through resonant thermo-tunneling favor faster carrier escapes [3] and near ideal carrier collection efficiencies, overcoming the large radiative losses encountered in traditional deep well devices. The presentation will discuss latest developments in dilute nitride quantum enhanced solar cells with a particular emphasis on the potential of advanced designs toward the realization of ultra efficient (50%) multi-junction devices.

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Renewable Energy Potential in Bulgaria – Some Computer Simulations Results

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The energy efficiency politics of the EU requires 11% till 2011 and 16% till 2020 of the Bulgarian electricity to be produced from renewable sources. An important aspect of the renewable energy is the reduction and the trade of CO_2 emissions, which determines its great ecological and economic importance and the need of better utilization of the country potential.

That is why the renewable energy resources of the country, including wind and solar energy, and their geographic pattern have to be well studied.

Detailed study of the wind and solar energy potential of the country – spatial distribution, temporal variation, mean and extreme values, fluctuations and statistical characteristics; evaluation from a point of view of industrial applicability is the objective of the present work.

The computer simulations were performed applying the 5th generation PSU/NCAR Meso-meteorological Model MM5 for years 2000-2007 with a spatial resolution of 3km over Bulgaria. Some evaluations of the country renewable energy potential, based on the simulation output are demonstrated in the paper.

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Energy Transfer between Two Filaments Due to Degenerate Four-Photon Parametric Process

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Recently energy exchange between two filaments crossing at small angle and with power slightly above the critical for self-focusing P_{cr} was experimentally demonstrated. In this paper we present a model describing the process of this transfer through degenerate four-photon parametric mixing. Our model confirms the experimental results that the direction of energy exchange depends on the relative transverse velocity (incident angle), laser intensity and initial distance between the pulses (relative initial phase). We also investigate the interaction between two collinear filaments in order to explain the filaments number reduction for powers close to P_{cr} in multi-filamental propagation.

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Modeling Interactions of Soliton Trains. Effects of External Potentials

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We outline the derivation of the Complex Toda chain (CTC) as a model describing the N-soliton train dynamics for the (scalar) NLS equations [1,2]

$$iu_t + \tfrac{1}{2}u_{xx} + |u|^2 u = V(x)u(x,t). \tag{1}$$

perturbed by external potentials of several types: a) harmonic potentials $V(x) = v_2 x^2 + v_1 x + v_0$, b) periodic potentials $V(x) = A\cos(\Omega x + \Omega_0)$ and c) shallow potential wells $V(x) = c_0(\tanh(x - x_{\mathrm{f}}) - \tanh(x - x_{\mathrm{in}}))$, $c_0 \ll 1$ and $x_{\mathrm{in}} < x_{\mathrm{f}}$.

The method can be generalized to treat the N-soliton train dynamics of the one-dimensional Gross-Pitaevsky equation, or the Manakov model in external potential

$$i\vec{u}_t + \frac{1}{2}\vec{u}_{xx} + (\vec{u}^{\,\dagger}, \vec{u})\vec{u} = V(x)\vec{u}(x,t). \quad (2)$$

of the same types as above.

We demonstrate that the perturbed CTC adequately models the soliton train dynamics for a wide region of the initial soliton parameters [1,2,3,4].

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A Non-Radial Model for Evaluating Multiperiod Aggregative Efficiency

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Most organizations or firms generate a time series of data that represent their performances in the resource utilization and output production over multiple periods. These organizations often desire an aggregated measure of efficiency for several periods. This paper provides a new slack based measure (SBM) model for evaluating multiperiod aggregative efficiency. The proposed model reduces the computational complexity of some recently introduced models. Furthermore, in addition to computing aggregative efficiency in multiple period, the proposed model computes the efficiency of each decision making unit (DMU) at each period. A real case study is given to show applicability of the proposed model.

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Generalization of Eulerian Numbers and Their Application

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In the paper a proposal of the Eulerian numbers generalization is presented. As an example of its application and, simultaneously, the source of inspiration we give the closed form formula for the nth derivative of a function which satisfies some differential equations with constant coefficients.

Keywords: Eulerian numbers, nth derivative

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Computer Modeling of High-Temperature Diffusion Processes

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Numerical methods selected for high-temperature diffusion process generalized model. The model is represented by a system of boundary value problems for the equations of heat conduction and diffusion,. When choosing method Different boundary conditions takes into account which reflect the nature of the various sources of heat. The proposed approach to the solution of nonlinear problems combined several techniques and allowed to realize a computer model of the high-temperature diffusion for a variety of tasks that are associated with the choice of the sintering mode and manufacturing of products from pressed powder with the required physical characteristics. The computer model is built to perform numerical

calculations in UML modeling as a class diagram of the implementation. Program developed based on the model allows calculations and build graphs of temperature distributions for products and molds, and graphs of the plasticizer concentration and the low-melting impurities in the product. The results obtained during the program can be used in an automated process control system of sintering pressing, sintering and heat treatment.

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Solutions of the Klein-Gordon Equation with Equal Scalar and Vector Harmonic Oscillator Plus Inversely Quadratic Potential

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The solutions of the Klein Gordon equation (KG) with equal scalar and vector harmonic oscillator plus inversely quadratic potential for s - waves have been presented using the Nikiforov-Uvarov method. The bound state energy eigenvalues and the corresponding un-normalized eigenfunctions are obtained in terms of the Laguerre polynomials.

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Iterative Methods for Generalized Riccati Equations on Open Source Software

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The broad applications of stochastic models lead to increased interest on generalized Riccati equations and their numerical solution. This report considers computer implementation of two iterative algorithms for finding the maximum solution of generalized Riccati equations. Lyapunov's iterative method and the linear matrix inequality method are interpreted in the programming environment of SCILAB and MATLAB and are applied in solving the presented problems. Numerical experiments were conducted to compare the their numerical behavior of the SCILAB and the MATLAB implementations.

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Metaplectic Quantization

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In this seminar, we give an overview on metaplectic quantization and apply the metaplectic correction on co-adjoint orbit. Also we present an explicit formula for inner product of half-forms with coefficients in Hilbert space of square integrable polarized sections of pre-quantum line bundle on co-adjoint orbit. Geometric Quantization program started by Kostant-Souriau Kirillov pre-quantization procedure, but it needed to technical geometric correction and more precisely it was not included in all constructions and K. Habermann by modifying Kostant-Souriau Kirillov's quantization, introduced Metaplectic Quantization. In fact, Orbit Method is a method to determine all irreducible unitary representations of a Lie group and Geometric Quantization is the physical counterpart of the Orbit Method. So, by this point of view, we see the more effectiveness of metaplectic quantization procedure on co-adjoint orbit.

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A Numerical Solution of Falkner-Skan Equation via a Shifted Chebyshev Collocation Method

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The Falkner-Skan equation is a nonlinear third-order boundary value problem defined on the semi-infinite interval $[0, 1)$. This equation plays an important role to illustrate the main physical features of boundary layer phenomena. This paper presents a new collocation method for solving the Falkner-Skan equation. The proposed approach is equipped by the orthogonal Chebyshev polynomials that have perfect properties to achieve this goal. The shifted Chebyshev-Gauss pints are utilized as collocation points. In addition, this method reduces solution of the problem to solution of a system of algebraic equations. Comparisons with other methods show that the proposed method is highly accurate and its convergence is very rapid.

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The Parallel Implementation of the Numerical Solution of the Inverse Problem of Boundary Function Recovery for the Tidal Models

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In this paper boundary function recovery in the problem of long-wave propagation in a large water area is considered.

The mathematical model of the shallow water equations on a spherical surface is used. A boundary of a numerical domain consists of a coastline ("hard") part and an open-water ("liquid") part. In general case the influence of the ocean through an open-water part of a boundary is uncertain. Therefore at "liquid" part of a boundary the boundary conditions contain a special unknown function which should be determined together with velocity and free surface level. Thus, the ill-posed inverse problem of recovery of the boundary function is considered.

To solve this problem we use additional information, e.g. observation of free surface level on a part of a "liquid" boundary. We investigate three different approaches to regularization of our ill-posed problem using adjoint operators and optimal control theory. For the numerical solving of the inverse problem the iterative algorithm is proposed which consists of alternate solutions of direct and adjoint equations and refinement equation of a boundary function.

To verify the iterative algorithm the numerical experiments of data recovery are carried out on the Sea of Okhotsk region. We use the model observation data of different smoothness – smooth, with white noise, with gaps.

To numerical solution the differential problems are reduced to algebraic ones by the finite element method (FEM). FEM permit to use the unstructured irregular meshes for computational domains of a complicated form but FEM is required a large amount of computations. Therefore the parallel software using MPI and OpenMP technologies is developed. Considerable attention is paid to the description, investigation and comparison of effective parallel numerical algorithms based on the MPI, OpenMP and MPI+OpenMP technologies.

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Control of Heat Source in a Heat Conduction Problem

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The mathematical model of thermal processes during the heat treatment of a moving axisymmetric environment, for example wire. is considered. The wire is heated by internal constantly operating heat source. It is presented in the form of initial-boundary value problem for the unsteady heat equation with internal constantly operating heat source. The purpose of the work is the definition of control parameter of temperature field of a moving area, which is heated by internal heat source. The control parameters are determined by solving a nonlocal problem for the heat equation. The problem of getting an adequate temperature distribution throughout the heating area is considered. Therefore, a problem of heat source control is solved, in particular, control by electric current. Control of the heat source allows to maintain the necessary, from a technological point of view, temperature in the heating area. In this paper, to find additional information about the source of heat the integral condition is used in the control problem. Integral condition, which is considered in the work, determines the energy balance of the heating zone and connects the desired temperature distribution in the internal points of area with temperatures at the boundaries. Control quality in an extremum formulation

of the problem is assessed using the quadratic functional. In function space, from a physical point of view, proposed functional is the absolute difference between the actual emission of energy and absorbed energy in the heating zone. The absorbed energy is calculated by solving of the boundary value problem. Methods of determining the temperature field of the control parameters are proposed. The resulting problem is solved by iterative methods. At different physical conditions, numerical calculations are carried out, control parameters of the heat treatment process are obtained.

Keywords: Nonlocal problem, integral condition, heat equation, control parameters

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Structural Methods of Solving Boundary Value Problems on the Basis of the Atomic Functions

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Structural method for solving boundary value problems involves construction of functions (making structures) as defined in the given area and geometrical exactly satisfy the boundary conditions of the boundary value problem. To construct such functions used mathematical tools to describe geometric objects on the basis of the theory of R-functions using the R-operations, the formation of which are carried out with the involvement of the atomic functions, techniques offered an approximate representation of R-operations and construction of complete structures.

We will investigate the boundary value problem for a differential equation $Au(x) = f$ considered in t-dimensional domain $\Omega \subset E^t$ with boundary $\partial\Omega$, $x = (x_1, x_2, ..., x_t) \in \overline{\Omega} = \Omega \cup \partial\Omega$; A – an elliptic differential operator of order $2m$, with the boundary conditions of the form $B_i u|_{\partial\Omega} = \sum\limits_{|s|<k_i} b_{is} D^s u = g_i$, $i = \overline{1,n}$; $k_i \leq k_{i-1} < m$, s – multi-index of order t, $s = (s_1, s_2, ..., s_t)$, $|s| = \sum\limits_{i=1}^{t} s_i$; $D^s u = \frac{\partial^{|s|}}{\partial x_1^{s_1} \partial x_2^{s_2} ... \partial x_t^{s_t}}$, b_{is}, g_i – given on the boundary $\partial\Omega$ functions with fully qualified differential properties.

Structural method for solving boundary value problem is directly related to the construction of a function $w = w(B_i, p_1, p_2, ..., p_n)$ called the structure of solutions that would be everywhere dense in a certain approximation spaces in terms of relevant metrics. As an approximation space can be considered, in addition to traditional spaces of polynomials, splines, etc., the space of linear combinations of compression and/or translations the atomic functions. Effective use of the structural method for solving boundary value problems of mathematical physics (R-functions method) is largely determined in ensuring supplies functional component structures, $p_1, p_2, ..., p_n$, pre-defined functional spaces, which must belong to solution of the problem.

Natural to expect that the set of structures, $W = \{w\}$, built in a constructive way that would ensure transparency establish approximation properties of the resulting structures. Building the structures of boundary value problems with calls for a precise mathematical description of the boundaries of the region to provide the necessary smoothness considered R-operations. Relevant R-operations can be built using the atomic functions, providing reception of infinitely differentiable R-operations used in the description of geometric areas. To solve this problem, consider the atomic function [the Rvachev's function] $up(x) \in C^\infty$, which has representation $up(x) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \exp(-itx) \prod\limits_{k=1}^{\infty} \frac{\sin t2^{-k}}{t2^{-k}} dt$, is an entire function of exponential type, and thanks to the Paley-Wiener theorem has compact support, i.e. $\operatorname{supp} up(x) = [-1, 1]$.

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Symmetry and Nonlinear Dynamics of High Spin Magnets

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The purpose of this report is finding nonlinear dynamic equations of magnets, the particles of which have the spin $s > 1/2$ and the exchange magnetic Hamiltonian has the $SU(2s+1)$ symmetry. In our study, we based on works [1,2], where the Hamiltonian approach has been developed for describing of the dynamics of a wide class of magnets. For simplicity, we have neglected of the influence of the crystal lattice and consider the magnets as a continuous medium. We have analyzed the magnetic degrees of freedom for normal and degenerate states of high-spin magnets, for which the Poisson bracket algebra has been obtained.

We have discussed the symmetry properties of the Hamiltonian and formulated the differential conservation laws. We have obtained flux densities of dynamic values for normal and degenerate states. Subalgebras of Poisson bracket of magnetic values for spin $s = 1/2; 1; 3/2$ have been established. We have been obtained non-linear dynamic equations of high-spin magnets with the $SO(3)$, $SU(4)$, $SU(2) \times SU(2)$, $SU(3)$, $SO(4)$, $SO(5)$ symmetry of exchange interaction. The connection between models of the magnetic exchange energy and the Casimir invariants has been discussed. The presence of several types of symmetry of exchange Hamiltonians and set of Casimir invariants lead to new possible magnetic states. The spectra of collective magnetic excitations taking into account relaxation were calculated.

The class of non-linear dynamic equations for high-spin magnets, which generalized the Landau-Lifshitz equation, has been obtained. The considered approach allows us to extend the dynamic theory of magnets for an arbitrary spin case considering the $SU(2s+1)$ symmetry. Non-linear solutions for this class of dynamic equations require their analysis and study.

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Modified Chebyshev Maps Based Pseudorandom Bit Generator

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In this paper we use Chebyshev polynomials of the first kind to generate random sequence. The modified pseudorandom bit generator use a new scheme of bit extraction applied to four Chebyshev maps. The properties of the generated sequence is analysed by NIST, ENT and DIEHARD tests.

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Long Range Filament as Vector Solitary Wave

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The broad-band spectrum ($\Delta k_z \approx k_0$) is one of the basic characteristics of the stable optical filament. The evolution of the so obtained filament can not be further described in the frame of the nonlinear paraxial optics, because the paraxial optics works correctly for narrow-band laser pulses only ($\Delta k_z \ll k_0$). The dynamics of broad-band pulses can be presented properly within different models of non-paraxial optics, such as UPPE models [1] or non-paraxial envelope equations [2]. Another standard restriction in the filamentation theory is the use of one-component scalar approximation of the electrical field $\vec{E}$. This approximation though, is in contradiction with recent experimental results, where rotation of the polarization vector is observed [3]. For this reason, in our investigations we use non-paraxial vector model up to second order of dispersion, and one generalized vector nonlinear polarization of the kind $\vec{E}_{\mathrm{nl}} = (\vec{E} \cdot \vec{E})\vec{E}$. We found Lorentz type solitary wave solution of the corresponding nonlinear system of equations, describing the optical filament as stable vector soliton.

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Global Behavior of the Solutions to Boussinesq Type Equation with a Linear Restoring Force

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We consider the nonexistence of global solutions to the Boussinesq type equation of the form

$$u_{tt} - \Delta u - \beta_1 \Delta u_{tt} + \beta_2 \Delta^2 u + m^2 u = \alpha \Delta(|u|^{p-1} u) \quad \text{for} \quad x \in \mathbb{R}^n, \ t > 0,$$

where $\beta_1 \geq 0$, $\beta_2 > 0$, $m \neq 0$ and $\alpha > 0$ are real constants, $1 < p < \infty$ for $n = 1, 2$, and $1 < p < (n+2)/(n-2)$ for $n \geq 3$.

We give sufficient conditions on the initial data with arbitrary high positive initial energy such that the solution blows up in a finite time or at infinity. The performed numerical experiments support the theoretical results.

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Finite Element Method for Fractional-Order Differential Equations

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In this work, we consider boundary value problems involving either Caputo or Riemann-Liouville fractional derivatives of order $\alpha \in (1,2)$ on the unit interval $(0,1)$. These fractional derivatives lead to non-symmetric boundary value problems, which are investigated from a variational point of view. The variational problem for the Riemann-Liouville case is coercive on the space $H_0^{\alpha/2}(0,1)$ but the solutions are less regular, whereas that for the Caputo case involves different test and trial spaces. The numerical analysis of these problems requires the so-called shift theorems which show that the solutions of the variational problem are more regular. The regularity pickup enables one to establish convergence rates of the finite element approximations. The analytical theory is then applied to the Sturm-Liouville problem involving a fractional derivative in the leading term. Finally, extensive numerical results are presented to illustrate the error estimates for the source problem and eigenvalue problem.

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Stochastic Resonance in a Generalized von Foerster Population Growth Model

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We study the stochastic dynamics of a population growth model with size-dependent carrying capacity. At intermediate values of population size the behavior of the deterministic counterpart of the model is similar to the von Foerster model for human population [1]. The influence of a fluctuating environment on the carrying capacity is modeled as a multiplicative dichotomous noise. Although for a similar model the probability density and the first moment of the population size in the stationary regime (i.e. in the long-time limit) has been investigated in detail [2], next to, nothing is known about the mean transition times between different noise-induced stationary states occurring in such systems in spite of their importance in ecological applications. Thus motivated, we have considered the mean duration of noise-induced transitions. Particularly, it is established that an interplay between nonlinearity and environmental fluctuations can cause single unidirectional discontinuous transitions (vs noise amplitude) of the mean population size, i.e. an increase of noise amplitude can induce a jump from a state with a moderate number of individuals to that with a very large one, while by decreasing the noise amplitude an opposite transition cannot be brought about. As our main result, we have found an analytical expression of the mean escape time for such transitions. It is shown that the mean transition time exhibits a strong minimum at intermediate values of the noise correlation time, i.e. the phenomenon of stochastic resonance occurs. Applications of the results in ecology are also discussed.

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Mathematical Theory of Ether

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The report presents the mathematical foundations of the unified fundamental physical theory, which only postulate is the postulate of the existence of the ether. It is shown that the basic equations and formulas of classical electrodynamics, quantum mechanics, the theory of electromagnetism and gravitation theory can be derived from the two nonlinear deterministic equations of dynamics of the ether as a continuous non-viscous medium in three-dimensional Euclidean space. These equations are derived from equations of classical Newtonian mechanics and they are invariant under Galilean transformations. Using the characteristics of the ether, which are its density and the velocity of propagation of the disturbances and fluctuations of the density, perfectly clear and consistent with the common sense definition of such physical categories as matter and antimatter, electric, magnetic and gravitational fields, the speed of light, photon, electron, proton, neutron, internal energy, mass, charge, spin, Planck's constant and the fine structure constant are given. The laws of Coulomb, Ampere, Biot-Savart-Laplace, Newton, the Maxwell and Dirac equations, the expressions for the Ampere and Lorentz forces and the values of the anomalous magnetic moments of the proton, electron and neutron are derived. An ether model of the hydrogen atom is constructed and it is proved that in addition to the ground and excited states the hydrogen atom can be in hydrino nonradiating states that can not be described by the Schrodinger equation and can explain the formation of dark matter in the Universe and the mechanisms of the low energy nuclear reactions (LERN).

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An Adaptive Collocation Approach for Nonlinear Initial Value Problems of Lane-Emden Type Equation Arising in Astrophysics

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Lane-Emden equations are nonlinear singular equations in the astrophysics that correspond to the polytropic models. They are categorized as singular initial value problems (IVPs) on the semi-infinite domain. In this paper, a new adaptive collocation method based on shifted Legendre-Gauss-Lobatto (ShLGL) nodes is proposed for solving nonlinear IVPs, which is valid for long-time calculations. The singular Lane-Emden type equations which have many applications in mathematical physics are then considered. In this method, the original IVP is replaced with a sequence of IVPs in mesh intervals. Then the IVPs in mesh intervals are step by step approximated using collocation. Convergence rate is investigated and it is shown that with both decreasing the mesh spacing and increasing the degree of the polynomial on each mesh interval, the accuracy can be improved. Some numerical examples are given and comparisons with other methods are made to demonstrate the efficiency and accuracy of the proposed method.

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Elastic Spirals

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In the present article we extend previous results on the elastic Sturmian spirals. New parametrizations are given in terms of the polar radius and the angle of inclination.

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Explicit Parameterisations of Willmore Surfaces

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The Willmore surfaces provide local extrema to the so-called Willmore functional, which assigns to each surface its total squared mean curvature.

There is a special class of axially symmetric Willmore surfaces whose graphs satisfy a relatively simple system of ordinary differential equations. The aim of this work is to give explicit parameterisations of the Willmore surfaces belonging to the foregoing class in terms of the Jacobian elliptic functions and elliptic integrals.

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Mathematical and Computational Modeling of the Diffraction Problems by Discrete Singularities Method

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The main objective of this study is reduced a boundary value problems of scattering and diffraction waves on plane-parallel structures to singular and hypersingular integral equations (SIEs and HSIEs). For this we use a method of the parametric representations of integral and pseudo-differential operators. Discretization of SIEs and HSIEs has been developed using the quadrature formulas of interpolation type. Computational modeling has been carried out with the help of discrete singularities method (DSM) for perfect electrically conducting or impedance pre-Cantor gratings. Numerical results of model scattering problems on periodic and boundary gratings and also on the gratings above a flat screen reflector are presented in this paper. The developed algorithm can be used for the numerical solution of a wide class of boundary value mathematical physics problems. Created programs allow analyzing solutions and advantages DSM for different diffraction structures.

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Using of 2D Padé Approximations in Shell Mechanics

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A method is proposed that provides a meromorphic continuation of the asymptotic polynomial solution of differential equations based on 2D Padé approximants in nonlinear shell theory. Calculations of displacements, stability and vibration of inhomogeneous loaded shells with developable principal surface by means of this method are represented. Accuracy of theoretic results are confirmed by means of holographic experiments with specimens made from stainless steel.

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Numerical Analysis of Nematic Liquid Crystals as Applied to Tunable Antennas

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In the current work we examine the application of Nematic Liquid Crystals (N-LCs) to frequency-agile antennas. Various patch antenna designs with a liquid crystal base are proposed. The N-LC microstructures are subject to a lowfrequency AC electric field. The most important property of N-LCs for applications is the fact that they are anisotropic and their electrical properties are determined by the macroscopic orientation of their molecules, i.e., the directors' tilt angles. However, these depend on the applied electric field, which means that they are tunable.

The above described problem is governed by a coupled system of PDEs. It is solved iteratively using a finite-difference scheme with relaxation. Once the director field is obtained, the dielectric properties of the material are determined for each value of the bias voltage. The proposed antennas are then simulated using HFSS. The return loss and resonant frequency are computed for each of value of the applied voltage. It is shown that the antennas under consideration can be tuned

using relatively low applied voltages. This demonstrates the potential of liquid crystal based antennas in tunable antenna design.

Keywords: Nematic liquid crystals, finite differences, frequency-agile antennas, effective dielectric constant

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Multicriteria Analysis of Ontologically Represented Information

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The work discussed during the presentation represents another attempt to design a decision support system for the software selection problem. The main idea is to utilize expert knowledge to help the user in selecting the best software / method / computational resource to solve a computational problem. We hope that, with the application of modern tools, such as ontological representation of domain knowledge and semantic data processing supported by multicriterial analysis, we will be able to develop a system that will efficiently support users. The context of the work is provided by the Agents in Grid (AiG) project, which aims at development of an agent-semantic infrastructure for efficient resource management in the (computational) Grid. Decision support within the system should help the user, without in-depth knowledge of available computational resources, to choose an optimal algorithm and/or resource to solve a problem from a given domain, and later to choose the best contract defining terms of collaboration with the provider of (just selected) resource(s) that are to be used to solve that problem.

The first evaluated method (discussed during AMiTaNS'2013) was the Analytical Hierarchy Process (AHP). This is a well known and widely applied method to approach complex problems, taking into account subjective assessment by multiple decision-makers. The aim of this year presentation will be to expand the scope of interest in potential decision support methods fitting our problem. Besides the AHP, will consider the TOPSIS method (Technique for order preference by similarity to ideal solution) supported by the ontological matchmaking. Moreover, we will inspect the PROMETHEE method that was designed as a group decision support method. While, above, we have mentioned three methods, during the presentation we will briefly analyze also other possible methods, e.g. the linear additive model

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Transformations Preserving Nonsingularity, Trace and Spectrum of Matrices

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The paper is devoted to discussion on the linear and nonlinear transformations preserving the nonsingularity, trace and spectrum of matrices. Goal of this research is to adapt the obtained results for testing numerical algorithms. Received results have also, undoubtedly, the scientific and didactic value. It seems that there is no such discussion, and the proper results either, in the literature, therefore we decided to correct this deficiency with this paper.

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Sensitive Response of a Model of Symbiotic Ecosystem to Seasonal Periodic Drive

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A N-species symbiotic ecosystem is studied by means of the Lotka-Volterra stochastic model with the generalized Verhulst self-regulation.

The effect of variable environment on the carrying capacities of populations is taken into account as an asymmetric dichotomous noise and as a deterministic periodic stimulus.

On the basis of Refs. [1, 2] we consider all species equivalent, so that the characteristic parameters of the ecosystem (or metapopulation) are independent of the species. With these assumptions, in the framework of the mean-field theory an explicit self-consistency equation for the system in the long-time limit is presented. Also, expressions for the probability distribution and for the moments of the population size are found.

In some cases the mean population size exhibits large oscillations in time, even if the amplitude of the seasonal environmental drive is small. The conditions for the appearance of such a phenomenon, caused by an interplay of a small periodic forcing and colored noise, are found and illustrated by graphs.

Particularly, it is shown that the occurrence of large oscillations of the mean population size can be controlled by noise parameters, such as amplitude and correlation time. Another important observation in ecological context is that the phenomenon gets more pronounced as the coupling strength of the symbiotic interaction increases.

Keywords: Verhulst self-regulation, symbiotic ecosystem, dichotomous noise, mean-field theory, sensitive response

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On One Numerical Scheme of the Solution of a Three-Dimensional Problem of Diffraction of an Electromagnetic Wave on Thin Ideally Conductive Screens

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One of ways of the solution of a boundary value problem of diffraction of the given primary electromagnetic wave on ideally-conductive screens is its reduction to a two-dimensional vector boundary integral equation with hypersingular kernel. Such integral is understood in sense Hadamard finite value. Right-hand of the integral equation is the projection of a vector of an incident wave to the tangential plane to the surface, and the unknown vector represents current on a body surface.

In the work the numerical scheme for the solution of such integral equation is considered. The surface cuts into cells with piecewise smooth border. On each cell unknown vector function is approximated by special function which in specific case of a flat cell degenerates into a constant vector. In a core of the integral operator the singular part is allocated. For singular part the integral can be reduced to contour integral on cell edge which in case of rectilinear sites of border of a cell can be calculated analytically. The integral of the rest is regular and can be calculated in the common ways, for example, through the integral sum. This scheme can be applied to a problem of diffraction on opened or closed surfaces enough arbitrariest form for which any geometrical symmetry a priori is not supposed.

Knowing approximate solutions of an integral equation, it is possible to find various characteristics of a secondary field. In particular, it is possible to find a field in a near zone; to find a field on infinity in the given direction (that allows to calculate the object Radar Cross Section (RCS)), to find the total reflected energy, etc.

This numerical scheme was verified both with use of known theoretical results, and in comparison with physical experiment. Comparisons of the surface currents on the cylinder, which length much more then diameter with theoretical values on the infinite cylinder were carried out. Dependence RCS of the sphere on a wave vector of an incident wave was constructed. Comparison of energy of the reflected field by the flat cylinder with theoretical value is carried out. Comparison of RCS of the terminating cylinder with experimentally values.

Results of numerical simulation allow to speak about satisfactory compliance with a diameter of partition making 1/7-1/10 from length of an incident wave or less.

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Numerical Simulation of the Dynamics of a Liquid Crystal in the Case of Plane Strain using GPUs

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Liquid crystals (mesomorphic materials) can be considered as micropolar continua, in which the elongated particles – domains of co-oriented molecules – can move at each point in accordance with the laws of dynamics of a viscous liquid and can rotate relative to this liquid, encountering viscoelastic resistance. Due to the unusual combination of properties of fluidity and elastic anisotropy, liquid crystals find wide applications in the data logging devices, displays, in the devices of processing and storage of information, in the medical and technical diagnostics, particularly in the thermography.

Under numerical analysis of deformation processes in liquid crystals the problems of ultrahigh dimension arise, because for correct computations it is necessary to coordinate steps of grids with small parameter of the model - characteristic linear size of the material microstructure. The methods of high-performance computing on multiprocessor systems of a cluster type and systems with GPUs are effective.

In this report the behaviour of a liquid crystalline medium is described on the basis of a simplified mathematical model, taking into account in the acoustic approximation the rotational degrees of freedom of the particles under the action of weak mechanical and thermal perturbations. Parallel computational algorithm is worked out by means of the Ivanov method for constructing of finite-difference schemes with controlled dissipative properties and the splitting method with respect to spatial variables. This algorithm takes into account the mutual influence of three multi-scale physical effects - transfer of acoustic energy due to translational motion, rotation of the particles under the action of tangential stresses and moments, and thermal diffusion. The algorithm is implemented numerically using the CUDA technology. The results of computations demonstrate the efficiency of proposed method and algorithm. Based on the Neumann-Richtmyer difference scheme, the Klein-Gordon equation describing the fields of tangential stresses and angles of rotation of the particles of a liquid crystal is solved numerically.

This work was supported by the Russian Foundation for Basic Research (grant no.14-01-00130) and the Interdisciplinary Integration Project no. 71 of the Siberian Branch of RAS.

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Equations of the Dynamics of a Liquid Crystal under the Influence of Weak Mechanical and Thermal Perturbations

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Liquid crystals have a wide range of applications due to the unusual combination of properties of fluidity and elastic anisotropy, which appear in a certain temperature range under weak external effects of different physical nature. One of the common approaches to constructing mathematical models for the description of thermomechanical behavior of a liquid crystal is based on its representation as a fine-grained continuum, in which the elongated particles - domains of co-oriented molecules - can move at each point according to dynamic laws of a viscous liquid and can rotate relative to this liquid, encountering viscoelastic resistance to rotation. The methodological basis for this approach is the work by the Cosserat brothers, where the equations of moment elastic continuum were written. The first model of a liquid crystal was proposed by Ericksen. The works by Aero, Leislie, Kalugin, Kondaurov are devoted to the construction of models by means of thermodynamical principles. Thus, currently a general system of equations is obtained, adequately describing a lot of qualitative features of the behaviour of materials in the mesomorphic aggregate state. However, for a detailed analysis of the processes, occurring in liquid crystals, it is appropriate to obtain more simple variants of the equations, describing the particular cases of motion. In this report the simplified model of a liquid crystal as a microinhomogeneous acoustic medium with rotating particles is constructed on the basis of assumption of the dependence of potential energy of elastic deformation on the volume change, the angle of relative rotation of particles, and the entropy. Instantaneous orientation of the molecules of a liquid crystal is taken into account. The thermal conductivity tensor is written considering the anisotropy, caused by the difference of coefficients of thermal conductivity in the direction along the axis of orientation of molecules and in the transverse direction.

This work was supported by the Russian Foundation for Basic Research (grant no.14-01-00130) and the Interdisciplinary Integration Project no. 71 of the Siberian Branch of RAS.

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Patch-Fitting on Surface Holes

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The problems associated with defining the surface shape of a complex deformable object such as a human figure are considerable. As yet, nobody has devised a system which is flexible enough to cater for the whole range of possible surface shapes of such a figure, while retaining the capability of displaying fine detail on the surface, and being sufficiently economical in use for it to be suitable for producing animations in a commercial situation. A lot of research focusing on developing schemes to produce surfaces with fine detail has been successfully carried out during the last three decades. However, most of these efforts fell some way short in addressing the problem of branching of surfaces.

Our work seeks to address this issue. Investigation of the properties of a variety of methods used to tackle the problem of single or multiple branching on a surface has been carried out. What emerged from this investigation was that there does not exist a completely satisfactory method at present, and that some element of compromise is involved in all approaches adopted to date. Additional investigation was necessary in order to derive a new method contributing to further development of complete complex deformable objects. The proposed method superimposes subdivision-surfaces on top of surfaces containing "holes," producing completely closed and smooth objects.

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Mathematics for Industry and Innovation in Europe

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In this talk, we will discuss the European service network of mathematics for industry and innovation, abbreviated EU-MATHS-IN (see www.eu-maths-in.eu). This is a **new initiative to boost mathematics for industry in Europe, and make the most of our expertise for a more efficient route to innovation**.

To leverage the impact of mathematics on innovations in key technologies it requires enhanced communication and information exchanges between the following categories of key players:

- R&D-staff in industry, science and society (i.e. the end-users of mathematical tools and expertises);
- Mathematicians in academia (i.e. the creators of novel mathematical tools and expertises);
- SMEs and spin-off companies with large mathematical content in their products and activities (i.e. the missing link between "creators" and "end-users" that transform mathematical innovations from academia into production-level software to be provided to industry);
- Furthermore, as the challenges to be dealt with are increasingly complex and multi-facetted, the integration on a European level of a large variety of mathematical expertises from different institutes is required.

EU-MATHS-IN aims to leverage the impact of mathematics on innovations in key technologies by enhanced communication and information exchange between and among the involved stakeholders on a European level. It shall become a dedicated one-stop shop to coordinate and facilitate the required exchanges in the field of application-driven mathematical research and its exploitation for innovations in industry, science and society. For this it shall build an e-infrastructure that provides tailored access to information and facilitates communication and exchange by player-specific sets of services. It will act as facilitator, translator, educator and link between and among the various players and their communities in Europe.

Besides a description of EU-MATHS-IN as an organization, the talk will also present several national initiatives to stimulate the use of mathematics. Both in the UK and The Netherlands, recently a survey was conducted by Deloitte showing that the contribution of mathematics can be up to 30 percent of gross value added. In several countries, books with success stories are being published. This is all very essential if we wish to foster the use of mathematics for industry and innovations. Several stunning examples will be given, showing a clear benefit when using mathematics at an early stage.

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Poincare Formalism for Describing of Radiation Damage Evolution

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Poincare formalism is applied for describing of radiation damage evolution. The system of nonlinear differential equations that describes evolution of radiation damage of materials under irradiation is considered. The microstructure change drives material properties and is a reason of radiation-induced effects: creep, swelling, and so on. The variables of the system which describe material under irradiation are densities of different radiation defects, defect characteristics, material properties and parameters of irradiation condition. We take into account nonlinear feedbacks between different elements of the model.

The qualitative analysis of this non-linear dynamical system is carried out. All critical points of the system, their stability and type are found. A phase portrait of the system is constructed.

The change of the phase portrait depending on the system parameters is investigated. All phase portraits of the system that are quality different are obtained. The bifurcation surfaces are obtained in the space of the parameters of the system (environment temperature, defect generation rate and so on). Physical interpretation of obtained results is given. Results of the experiments and simulation are compared.

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Analogue of Gauss-Lucas Theorem for Complex Polynomials

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Let $S(\phi) = \{z : \ |\arg(z)| \geq \phi\}$ be a sector on the complex plane $\mathcal{C}$. If $\phi \geq \pi/2$, then $S(\phi)$ is a convex set and, according to the Gauss-Lucas theorem, if a polynomial $p(z)$ has all its zeros on $S(\phi)$, then the same is true for the zeros of all its derivatives. The main theorem in this lecture, called **Sector theorem**, asserts that if the polynomial $p(z)$ is with real and non negative coefficients, then the same is true also for $\phi < \pi/2$, when the sector is not a convex set on the complex plane.

The Sector theorem is applied to prove stronger Rolle's theorem for complex polynomials.

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Numerical Simulation of Separated Flow over Three-Dimensional Complex Shape Bodies with Some Vortex Method

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Vortex methods are among possible utilizations of the method of discrete singularities in appliance to aerohydrodynamics. Authors present the improved numerical vortex frames scheme for simulating the 3D low-speed subsonic unsteady flows over bodies with complex geometry. It is assumed that vortex trace appears on predefined separation lines and consists of 'near' and 'far' zones. The body surface and 'near' zone of vortex trace are modeled with closed vortex frames and 'far' zone is approximated with isolated vortex segments. Usage of isolated vortex frames provides opportunities to simulate how vortex trace rounds obstacles and lets vortex structures to self organize.

Unknown intensities of vortex frames that model the body surface are found on each step of integration in time from system of linear equations that approximates the boundary integral equation. This integral equation represents the boundary condition of no flow penetration through the body surfaces an written in such way that allows to simulate both thin and solid bodies and also bodies with complex geometry consisting of solid and thin components.

Transformation of vortex structures is performed on each step of integration in time and from algorithmic point of view appears to be special case of the task of N bodies. To fasten calculations authors use the method of mosaic-skeleton approximations of big matrices developed by E.E. Tyrtyshnikov (Institute of Numerical Mathematics of Russian Academy of Science). This method allows to significantly fasten the matrix-vector multiplication operation.

To calculate pressure field the numerical scheme based on the discretization of analogue of Cauchy-Lagrange integral for vortex flows is used. The integral was presented in paperwork of G.Ya. Dynnikova (Moscow State University).

The following results are presented: calculations of aerodynamic model problems, numerical modeling of flow past whole body of modern medium-range passenger aircraft, calculations of aerodynamic characteristics of the aircraft. The results of calculation of pitching and side aerodynamic characteristics are compared to experimental data got in TsAGI (Central Aerohydrodynamic Institute).

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Semi-Lagrangian Approach in Finite Element Method for Navier-Stokes Equations of Viscous Heat-Conducting Gas

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The approach is proposed for numerical solving the Navier-Stokes equations for two-dimensional motion of viscous heat-conducting gas. The discretization of equations is performed by a combination of a special semi-lagrangian method for transport derivatives and the conforming finite element method with piecewise linear basis functions for other terms. This method gives a more simple structure of the discrete system of equations and has no restriction on relation between temporal and spatial mesh-sizes similarly to the Courant one. The results of numerical studies of the structure of a supersonic flow around an obstacle for a wide range of Mach and Reynolds numbers will be presented.

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Strouhal Number Analysis for a Karman Vortex Gas Flow past a Square in a Microchannel at Low Mach Number

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The rapidly emerging industry of micro-electro-mechanical devices gives rise to new potential microfluidic applications. The analysis of the possible flow regimes is an important task of any microfluidic investigation. For a gas flow the transition between steady and unsteady regimes occurs at small Knudsen number Kn < 0.1 (Kn $= \ell_0/L$, where ℓ_0 is the mean free path of the gas molecules and L is the characteristic length). A continuum approach based on the Navier-Stokes-Fourier equations is applicable for this investigation. On the other side, the microfluidic problems include non-equilibrium effects to be taken into account. In this paper we compare Strouhal number obtained from molecular (DSMC method) and continual (SIMPLE-TS algorithm) approach, for a flow past square in a microchannel at transition from steady to unsteady flow regime for a fixed Mach number (M $= 0.4$).

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Application of the Homotopy Analysis Method for Solving the Two-Dimensional Steady-State Heat Conduction Problem

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In this paper we propose an application of the homotopy analysis method for solving the two-dimensional steady-state heat conduction problem. Discussed method is based on the concept of creating the function series. The paper presents the sufficient condition for convergence of this series and the error estimation of approximate solution obtained by using the partial sum of received series. Examples illustrating the usage of investigated method are also presented in the paper.

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Using Circle Map in Pseudorandom Bit Generation

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We propose a novel Circle map based pseudorandom bit scheme. We evaluated the output streams by the NIST, DIEHARD and ENT statistical packages. The results of the test show that the new generator provides an exclusive level of randomness of the bit sequences.

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Some Estimates Bellow the Modulus of Some Integrals of the Unit Circle

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In this paper, we make some estimates bellow the modulus of some integrals of the unit circle in the complex plane.

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Dynamic Fracture of Functionally Graded Magnetoelectroelastic Composite Materials

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The stress, magnetic and electric field analysis of multifunctional composites, weakened by impermeable cracks, is of fundamental importance for their structural integrity and reliable service performance. The aim is to study dynamic behavior of a plane of functionally graded magnetoelectroelastic composite with more than one crack. The coupled material properties vary exponentially in an arbitrary direction. The plane is subjected to anti-plane mechanical and in-plane electric and magnetic load. The boundary value problem described by the partial differential equations with variable coefficients is reduced to a non-hypersingular traction boundary integral equation based on the appropriate functional transform and frequency-dependent fundamental solution derived in a closed form by Radon transform. Software code based on the boundary integral equation method (BIEM) is developed, validated and inserted in numerical simulations. The obtained results show the sensitivity of the dynamic stress, magnetic and electric field concentration in the cracked plane to the type and characteristics of the dynamic load, to the location and cracks disposition, to the wave-crack-crack interactions and to the magnitude and direction of the material gradient.

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Influence of NO_x Emission on WRF-CMAQ Regional Modeling System Results

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The air quality modeling system WRF-CMAQ was applied to the European domain for the year 2010 in the frame of the Air Quality Model Evaluation International Initiative (AQMEII), Phase 2. The model system was set up for a domain

of $5000 \times 5000 km^2$ size with horizontal resolution of 25km.The emissions at European level were available through AQMEII and further processed in a way to feed the chemistry transport model CMAQ. Two NOx scenarios take place -– full emissions and emissions reduced by 30%. The meso-meteorological model WRF was driven by NCEP GFS data with $1^\circ \times 1^\circ$ resolution. The chemical boundary conditions were extracted from MACC global simulation data. Model performance was investigated by means of AQMEII-2 web based evaluation platform. A preliminary model evaluation for ozone, nitrogen dioxide and particulate matter was conducted based on comparison between simulated and observed concentrations at different type of surface stations in the EU wide domain. Model performance with the reduced NO_x scenario was characterized by high overestimation for ozone and underestimation for the other pollutants. The full NO_x emission scenario also shows ozone overestimation, results more close to the measurements. The relative statistical indicators were discussed also in view of recently published performance criteria.

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Carreau Model for Oscillatory Blood Flow in a Tube

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The analysis of the blood flow dynamics (hemodynamics) in tubes is crucial when investigating the rupture of different types of aneurysms. The blood viscosity nonlinear dependence on the flow shear rate creates complicated manifestations of the blood pulsations. Although that there exists a great number of studies, experimental and numerical, this phenomenon is still not very well understood. The aim of the present work is to propose a numerical model of the oscillatory blood flow in a tube on the basis of the Carreau model of the blood viscosity (nonlinear model with respect to the shear rate). The obtained results for the flow velocity and tangential stress on the tube wall are compared well with the existing experimental results.

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A Nonclassical Radau Collocation Method for Solving the Lane-Emden Equations of the Polytropic Index $4.75 \leq a \leq 5$

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A numerical method for solving the Lane-Emden equations of the polytropic index when $4.75 \leq a \leq 5$ is introduced. The method is based upon nonclassical Gauss-Radau collocation points and Freud type weights. Nonclassical orthogonal polynomials, nonclassical Radau points and weighted interpolation are introduced and are utilized in the interval $[0,1]$. A smooth, strictly monotonic transformation is used to map the infinite time domain $x \in [0,1)$ onto a half-open interval $t \in [0,1)$. The resulting problem on the finite interval is transcribed to a system of nonlinear algebraic equations using collocation. The method is easy to implement and yields very accurate results.

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Numerical Solutions of Physical Models for Propagation of Ultrashort Light Pulses

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Numerical models describing propagation of ultrashort light pulses at various physical conditions have been considered and solved numerically. At light pulses of intensity below ionization “threshold pulse propagation within (3+1)D nonlinear Schrödinger equation is investigated in low dispersion media. A new pulse propagation regime, called optical tsunami, is found. For high-intensity case, a physical

model based on (3+1)D nonlinear envelope equation is described and solved numerically. Ionization contribution to group velocity dispersion is introduced in the model. Pulse compression and stable pulse propagation of the compressed pulse is found for the first time at realistic physical conditions.

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The Power of Trefftz Methods: From Finite-Difference to Discontinuous Galerkin Schemes and from Macromolecules to Metamaterials

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In mathematical physics and engineering, Trefftz approximations by definition involve functions that satisfy the underlying differential equation of the problem. Examples include harmonic polynomials for the Laplace equation; plane waves, cylindrical or spherical harmonics for wave problems, and so on. Trefftz approximations are well established in the context of pseudo-spectral and domain decomposition methods, but this talk calls attention to applications that are known less well or are entirely new:

- Finite difference-Trefftz schemes of arbitrarily high order. They are obtained by replacing the classical Taylor expansions with local Trefftz approximations.
- Numerical and analytical asymptotic boundary conditions based on Trefftz approximations.
- Boundary-difference Trefftz methods that are analogous to boundary integral equations but are completely singularity-free.
- Discontinuous Galerkin–Trefftz methods for Maxwell's electrodynamics.

- Applications to the electrostatics of macromolecules (the Poisson-Boltzmann equation).
- Homogenization of electromagnetic and photonic metamaterials: a two-scale theory involving Trefftz approximations on both coarse and fine levels. This explains, in particular, "optical magnets" – artificial magnetism at high frequencies.

This discussion of the versatility and power of Trefftz methods is intended to stimulate their application in many other areas of applied science and engineering.

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Modeling the Dynamics of Vortex Wake in Temperature Inversion Conditions

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The report presents the results of numerical simulation of vortex wakes evolution behind aircraft under ground atmospheric temperature inversion conditions.

As an example a low-altitude flight of B-747-type aircraft was considered. The cruising altitude was chosen to show that vortex wake dynamics is determined by viscous interaction with ground boundary layer. It is known that at low heights vortex wake generates separation of ground boundary layer that results in a vortex wake rise to a certain height. The presence of crosswinds lead to asymmetry in the vortex bundles evolution.

The numerical algorithm is based on the discrete vortex method and takes into account additional damping of vortex wake caused by stratification. Also, lowering velocity variation of the vortex wake due to temperature stratification is taken into account. Vortex wake viscous interaction with the underlying surface is counted by means of secondary vortices generation modeling the ground boundary layer separation.

Temperature profiles of the ground atmosphere layer obtained experimentally with the help of microwave radiometer MTP-5 were used in the calculation.

Three cases – ground inversion, raised inversion and raised inversion with ground isotherms – have been examined. In addition, the influence of crosswinds with varying intensity on vortex wake dynamics in inversion conditions was considered.

It was shown that raised inversion has a significant impact on the lift height of the vortex wake caused by its interaction with the separated ground boundary layer.

This work was supported by the Russian Foundation for Basic Research, Projects 12-07-00697, 13-07-00276.

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Numerical Simulation of the Aluminum Production

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In mathematical modeling of aluminium manufacturing processes, the main object of studying is an aluminium electrolysis cell. Computational techniques are applied to solve the following problems:

- For developing a new cell design. On the basis of theoretical-computational researches, new technical solutions are prepared for all basic structural elements of the cell;

- To modernize and improve constructions. Within the framework of the existing production technology, simulations of critical units of electrolyzers are performed;

- For optimization of technology. To increase technical and economic indices of production, it is necessary to optimize control algorithms using an extended diagnostic base for the electrolyzer state and employing dynamic mathematical models for the investigation of cells.

As a technical system, an aluminium cell is a complex object for mathematical modeling. This is due to the complexity of the physical and chemical phenomena that occur in the cell; They are essentially interconnected. To predict electrolytic cells, emphasis is on describing the thermal state of the cell, which is governed by the existing current. Thermal and electrical balances of the cell are calculated using the full 3D equations of heat transfer and electrodynamics of continuous media. Unfortunately, we face large uncertainties in the parameters used in these mathematical models.

In modern electrolytic cells, currents form a powerful electromagnetic field, which creates conditions for the development of MHD instabilities. On the basis on mathematical modeling of the dynamics of an electrolyte and molten metal in electromagnetic fields, efficient technical solutions are obtained for bus bars of the cell.

Quick diagnosis of the current state of the cell is conducted primarily via measuring its electrical resistance. Nowadays, new technical possibilities are studied to extend the base of electrical measurements, i.e., to implement dynamic current measurements for individual anodes. Indirect measurements of currents through anodes are based on the observation data processing for the electric potential at points of the anode frame by means of solving the corresponding inverse problems.

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A Numerical Investigation of Stability of 1D Soliton Solutions of Boussinesq Paradigm Equation in the 2D Case

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The different versions of Boussinesq equation (BE) model surface waves in shallow fluid layer. One important feature of BE is the balance between the nonlinearity and dispersion, which leads to solutions of type of permanent waves (solitons).

Here we investigate numerically the time evolution and stability of some known 1D soliton solutions of Boussinesq paradigm equation in 1D and in a 2D setting. A moving frame coordinate system helps us to keep the structures in the center of the coordinate system, where the grid is much finer. It also reduces the effects of the reflection from the boundaries, allows us to use a small computational box and to compute the solution for very large times.

The numerical experiments show that the stable 1D solutions preserve themselves for very large times. The corresponding solutions of the 2D problem for the same parameters and in small intervals for y also preserve their shape for very large times.

But the solutions of the 2D problem in large intervals for y seem to be not stable – the waves preserve their shape in relatively long intervals of time (depending on the parameters), but after that the waves lose their constant behavior in the $y-$direction, the solutions start to grow and blow-up. The number of the maxima, which appear in the $y-$direction, strongly depends on the size of the domain in this direction, as well as on the problem's parameters.

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Mathematical Modeling of Heat Transfer Problems in the Permafrost

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Research of changes of soil temperature regime is a necessary element of the geotechnical studies for the construction objects in regions of permafrost soils occurrence. For seasonal thawing of frozen soil the physico-mechanical characteristics are changed, which leads to the violation of their bearing capacity. For numerical simulation of the process mathematical model of heat and mass transfer is constructed taking into account the freezing temperature of the liquid in the seasonal cooling devices and typical outside temperature for Yakutia (from -55°C to +35°C). Mathematical model of the process is described by a nonstationary heat equation with phase transitions of pore water. The numerical realization of the problem is based on the finite element method using a library of scientific computing FEniCS.

In this work we present results of numerical simulation of three-dimensional temperature fields in soils for various applied problems: the railway line in the conditions of permafrost for different geometries, the horizontal tunnel underground storage and greenhouses of various designs in the Far North. The calculations were performed on a computational cluster of North-Eastern Federal University.

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On Certain Approximation Problem in Normed Spaces

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The paper is a continuation of research undertaken in Cater's paper [1].

Let $(\mathbb{X}, \|\cdot\|)$ be the normed space. We have given the sequence of elements $\{x_n\}_{n=1}^{\infty} \subset \mathbb{X}$ and $x \in \mathbb{X}$ of the following property

$$\forall \varepsilon > 0 \; \exists \{x_{n_i}\}_{i=1}^{k} : \|x - \sum_{i=1}^{k} x_{n_i}\| \le \varepsilon. \tag{1}$$

It is known that this condition does not guarantee the existence of an infinite subsequence $\{x_{n_i}\}_{i=1}^{\infty}$ such that

$$\sum_{i=1}^{\infty} x_{n_i} = x. \tag{2}$$

One of the main goals of our paper is to give the additional conditions which, together with (1), would be sufficient for approximating element x by means of the series as in (2).

References

[1] F.S. Cater (1984) Differentiable, nowhere analytic functions, *Amer. Math. Monthly* **97**, 618–624.

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Ac-Driven Nonlinear Schroedinger Equation and Double Sine-Gordon Equation: Numerical Study of Complexes of Localized States

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We investigate complexes of localized states in two dynamical systems: (i) directly driven nonlinear Schroedinger equation (NLS) and (ii) double sine-Gordon equation (2SG). Both systems have a wide range of physical applications. Our numerical approach is based on the numerical continuation with respect to the control parameters of the quiescent (stationary)solutions and stability and bifurcation analysis of the linearized eigenvalue problem. Multisoliton complexes of the NLS equation are studied in the undamped and the weak damping regimes. We show that in the weak damping case the directly driven NLS equation holds stable and unstable multi-soliton complexes. Also, undamped ac-driven two-soliton complexes can stably travel with constant velocity. The results are confirmed by means of direct numerical simulations of the time-dependent NLS equation. Properties of the multi-fluxon solutions of 2SG equation are studied depending on the parameter of the second harmonic. We show that the second harmonic changes properties and increases the complexity of coexisting static fluxons of 2SG equation. The results are discussed within the frame of the long Josephson junction model.

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Numerical Simulation of Gas Dynamics and Heat Exchange Tasks in Fuel Assemblies of the Nuclear Reactors

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This report presents a PC-based program for solution gas dynamics and heat exchange mathematical tasks in fuel assemblies of the fast-neutron nuclear reactors. A fuel assembly consisting of bulk heat-generating elements, which are integrated together by the system of supply and pressure manifolds, is examined. Spherical heat-generating microelements, which contain nuclear fuel, are pulled into the heat-generating elements. Gaseous coolant proceed from supply manifolds to heat-generating elements, where it withdraws the nuclear reaction heat and assembles in pressure manifolds. Gas stream in heat-generating elements is considered to be axisymmetric and consequently is two-dimensional, while in the manifolds it used to be one-dimensional. For clarification of the coefficients of resistivity in pressure manifolds, I present a PC-based program for calculation of two-dimensional axisymmetric stream of the incompressible in viscid gas. In this case, a gas stream in a pressure manifold is considered to be vertical and its calculation is conducted with high-performance methods of discontinuous vortices.

The shell of the heat-generating element consists of two coaxial cylinders. Spherical heat-generating microelements are pulled into the free-space between them. Heat-generating elements of the axial pumping are being reviewed, that is why the bottoms of the cylinders are transparent. I use cylindrical coordinates, so the heat-generating elements naturally splits up into elementary volumes. As the average flow in the heat-generating elements is axisymmetric, we sufficiently can examine one axial layer of such volumes. Integral relations, which result from the laws of conservation of mass, impulse and total internal energy of heat-transfer agent, are compiled for each elementary volume. Along with this, in each primitive cell, we also consider heat balance equations for elementary volumes in the heat-generating microelement, which is a sample of a spherical filling. The building method of the finite-difference scheme provides its persistence on mass velocity, convection component, complete internal energy in heat-transfer agent and thermal energy in the heat-generating microelements. Impenetrable walls of the collectors are assumed to be smooth and axisymmetric.

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